MATHEMATICS

Note to Freedom School teachers:

This diagnostic is rather important in the summer curriculum. Your students will demonstrate a wide variety of levels of mathematical knowledge and we wish to satisfy individual needs as much as possible. This test should help you find just what each student needs. Administer the test in the manner you think best. Some students will only be able to solve a few of the arithmetic problems, if that. Let them go when they feel they can do no more. I would suggest a very liberal time allowance. Most likely few of the students have seen any set theory but, given time, many could solve that section. Perhaps you'll want to break the test up into two parts. Perhaps it would be best for some students if you sat down with them and went through the test problem by problem, offering hints when necessary. Those students who are able to work the test will want something new. Suggestions are analytic geometry, probability theory, or the binary number system. But you are invited and encouraged to use your imagination in inventing a course for them. One work of advice. The standard method of teaching math in Mississippi is through routine drill, and more routine drill. If your course tends to seem routine, like regular school, the students will tend to lose interest and you may lose them. Be creative. Experiment. The kids will love it.

Diagnostic in Mathematics for High School

Arithmetic

Add a) $\begin{array}{r} 57 \\ \underline{76} \end{array}$ b) $\begin{array}{r} 579 \\ \underline{638} \end{array}$ c) $\begin{array}{r} 78 \\ 79 \\ 65 \\ 6 \\ 103 \\ 12 \\ \underline{49} \end{array}$ Subtract $\begin{array}{r} 7183 \\ \underline{-4897} \end{array}$

Multiply 324 by 123. Solve: $437\overline{)486,735}$

Solve the following by "short" division: $7\overline{)53,879}$

Fractions

Solve as indicated

a) $\frac{7}{10} + \frac{2}{10} =$

b) $\frac{7}{8} - \frac{1}{4} =$

c) $\frac{2}{3} + \frac{4}{7} =$

d) $\frac{7}{9} - \frac{1}{10} =$

e) $4\frac{2}{3} + 7\frac{1}{10} =$

f) $\frac{2}{3} \times \frac{4}{7} =$

g) $\frac{1}{2} \times 4\frac{2}{3} =$

h) $8 \times 4\frac{2}{3} =$

i) $\frac{4}{3} \div \frac{4}{3} =$

j) $\frac{1}{2} \div 4\frac{2}{3} =$

"Signed numbers"

Solve a) $\begin{array}{r} 14 \\ \underline{-7} \end{array}$ b) $\begin{array}{r} 7 \\ \underline{-14} \end{array}$ c) $\begin{array}{r} -7 \\ \underline{-14} \end{array}$ d) $-8 \times 7 =$ e) $-16\overline{)5840}$

Write in decimal form: a) $\frac{1}{5} =$ b) $1\frac{3}{4} =$ c) $\frac{2}{3} =$

Write as a fraction: a) $.25 =$ b) $1.4 =$

Algebra

Solve

a) $Y \times Y =$

b) $Y^2 \times Y^3 =$

c) $Y^3 \times Y^{-2} =$

d) $Y^{-3} \times Y^2 =$

e) $(Y^3)^2 =$

f) $Y \times Y^{-1} =$

g) $Y^4 \div Y =$

h) $\frac{Y^{17}}{Y^4} =$

Polynomials

Solve

a) $$\begin{array}{r} Y^2 + 4Y - 32 \\ \underline{+\ Y + 8} \end{array}$$

b) $(Y^2 + 4Y - 32) - (Y + 8) =$

c) $(Y^2 + 4Y - 32) \times (Y + 8) =$

d) $Y + 8\overline{)Y2 + 4Y - 32}$

We know that $(Y + 2) \times (Y - 1) = Y \;\; + Y - 2$. Find two expressions which when multiplied together give $Y^2 + 5Y + 6$.

Find two more which give $Y^2 - 25$.

Find two more which give $Y^2 + 2Y + 1$.

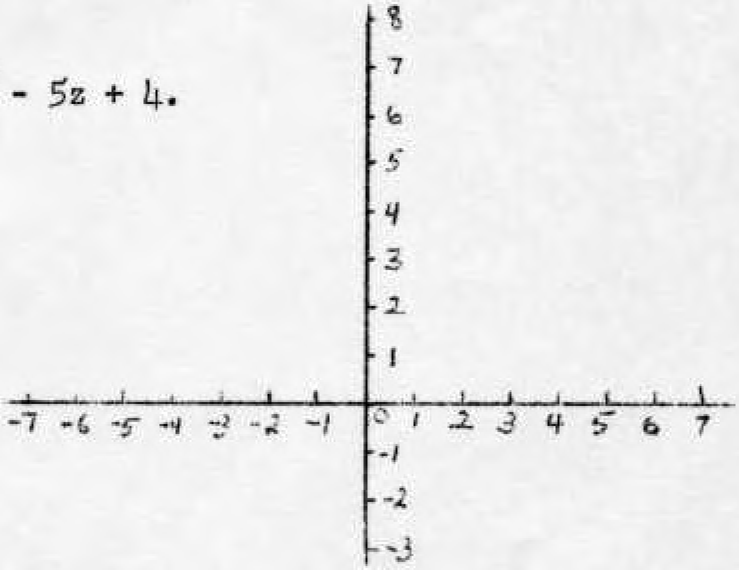
Find Y if $3Y - 2 = Y + 4$.

Find Z if $Z^2 + 4Z - 5 = 0$.

Find Y and Z if:
$$\begin{aligned} Y + Z &= 5. \\ Y - 7Z &= -3. \end{aligned}$$

What two numbers can be added together to give 8 and subtracted from one another to give 3?

Graph the function $y = z^2 - 5z + 4$.



Find the slope and the y-intercept of the equation which passes through (-1, 1) and (5, 3). (See graph on page above.)

Set Theory

A and B are <u>sets</u>, or collections of objects or terms. $C = A \cap B$ is the set of things in both A and B. $D = A \cup B$ is the set of things in either A or B, or both.

Example: If A is /2,4,6,8/   C, or $A \cap B$ is /6/
                   B is /3,6,9/   D, or $A \cup B$ is /2,3,4,6,8,9/

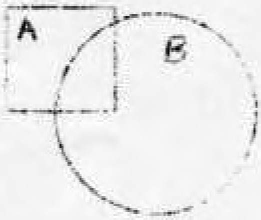
If A is /w,x,y,z/, B is /r,s,v,x/, find C, or $A \cap B$

D, or $A \cup B$

If A is all whole numbers greater than 1 and B is all whole numbers less than 6 find:

$A \cap B$ $A \cup B$

A is the area inside the square and B is the area inside the circle. Draw vertical lines in $A \cap B$ and horizontal lines in $A \cup B$.



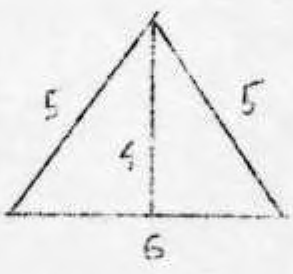
Geometry

How many lines can be drawn between two points?_______

Two non-parallel lines meet in how many points?_______

What is an angle?

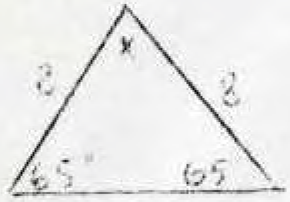
Find the area and perimeter of the rectangle and the triangle.



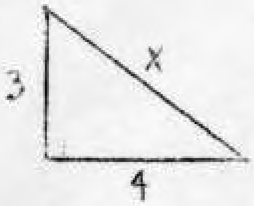
Find the value of x in the following figures:

a) x =

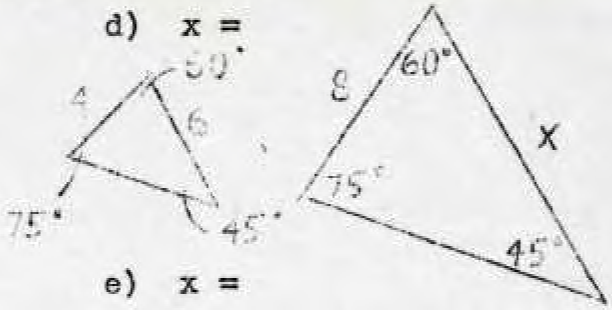
b) x =



c) x =



d) x =



e) x =

Supplementary Lectures

Note to teachers: these lectures are intended to give a bit of mathematics from a different point of view. You may alter or add to this as you like. (Actually these are only a couple of ideas of how a teacher can show his students something new and different. You will need to amplify what is written here.) Feel free to write up your own lectures.

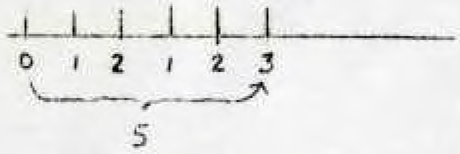
Lecture 1, Geometric Computation.

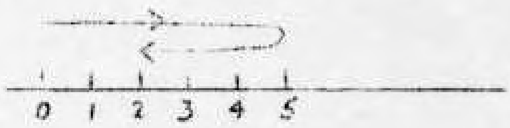
The point of this lecture is to demonstrate methods of addition, subtraction, multiplication, division, and the taking of square roots by geometric methods. Recall that the square root of a number Y is the number Z such that Z times Z equals Y. The square root of 9 is 3 because 3 times 3 equals 9.

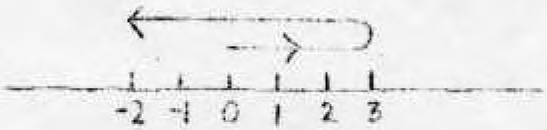
Addition and subtraction are rather easy. First draw a horizontal line. Call it an axis and find a point on it which we can call the "origin." Open the compass to a given unit lenth. Let's add 2 and 3. First place the point of the compass on the origin and mark off a unit length on the axis to the right of the origin. Then place the point of the compass on the new point and mark off another point to the right which will be a unit length's distant. This gives us our two. Mark off three more places to the right. Now we've added 2 and 3 and are 5 units to the right of the origin which illustrates that 2 + 3 = 5.

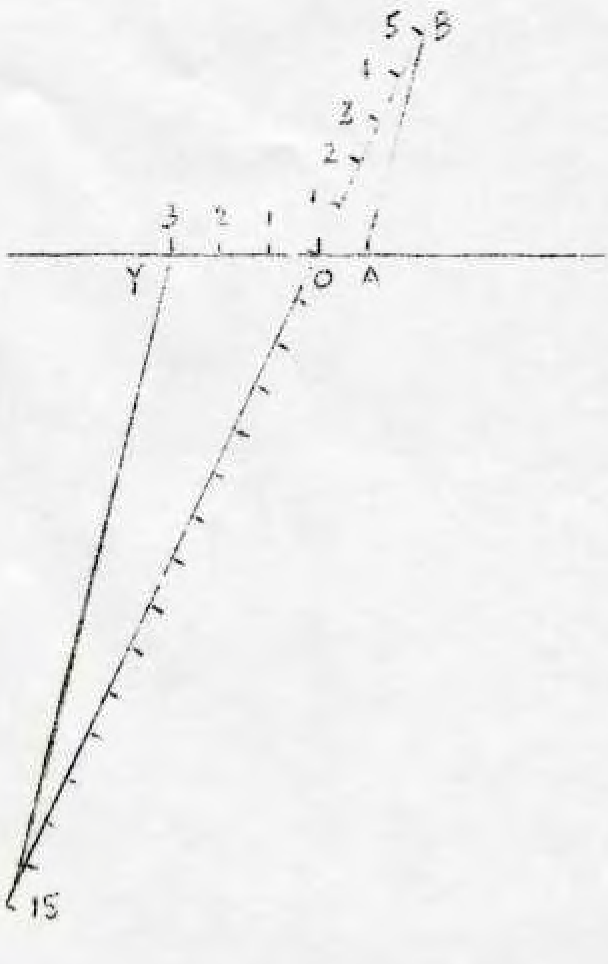
Subtraction is similar. Let us subtract 3 from 5. Mark off 5 units to the right of the origin. Then we mark off 3 units back toward the origin. We are now 2 units to the right of the origin which illustrates that $5 - 3 = 2$. We can take 5 from 3. We wind up two units to the <u>left</u> of the origin which illustrates that $3 - 5 = -2$. This illustrates how negative numbers arise.

Multiplication is another matter. First mark off a unit length to the right of the origin. Let's multiply 3 by 5. Next mark off 3 units to the left of the origin. Draw a line through the origin. Mark off 5 units in one direction and connect the 5 mark with the unit mark. Then draw a line through the 3 mark parallel to the line we just drew. It should hit the second line through the origin at a point 15 units away from the origin, if our measurement has been accurate. This illustrates that $3 \times 5 = 15$.









This can be explained by the fact that figures with the same shape have corresponding sides in equal proportion. Take two squares, one with side 7 and another with side 5; then all sides are in the ratio 7:5 with the corresponding side. The same holds for triangles. The triangles we drew have the same shape because the angles can be shown to be equal. (The teacher can show this by means of lines through parallel lines and opposite angles of two intersecting lines.)

Then one side is 3 units long and the corresponding side is 1 unit long. Our unknown side corresponds to the side 5 units long. Therefore it is 3 times as big and is 15 units long.

Division is similar. To divide 15 by 3, mark the 1 line and the 3 line as before, then draw a line through the origin. Mark off 15 units and connect the 15 mark with the 3 mark. Draw a parallel line through the 1 mark. It should hit the second line through the origin 5 units from the origin. This illustrates that 15/3 equals 5.

Now to find the square root, say of 9. Mark off 9 units to the right of the origin and 1 unit to the left of the origin. We how have a line 10 units long, the middle of which is 4 units to the right of the origin. Draw a circle with radius 5 units and center at the middle of our line. Draw a line through the origin perpendicular to the axis. It should intersect the circle at a point 3 units from the axis. Thus we have found the square root of 9 which is 3.

$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$

2. Divisibility and Various Methods of Addition

Often times we can tell which numbers divide a given number just by looking at it. For instance, if a number ends in 0 we know it is divisible by 10, if it ends in 5 we know it is divisible by five, if it is even, it is divisible by 2. We can also tell if it is divisible by 3, 9, 11.

Add up the digits. If they add up to a multiple of 3, then the number is divisible by 3. If they add up to a multiple of 9, then the number is divisible by 9. Of course such a number is divisible also by 3. Take 495. $4 + 9 + 5 = 18$. $3 \times 6 = 18$ and $2 \times 9 = 18$ so we know that 495 is divisible by 3 and 9. Sure enough, $165 \times 3 = 495$ and $55 \times 9 = 495$. Now let's add the first and last digits. $4 = 5 = 9$, the middle digit. For any 3 digit number, if this is true, the number is divisible by 11. But not all 3 digit numbers divisible by 11 have this property. Take 902. But if we add the outer digits and subtract 0, we get 11. Thus we know that 11 divides 902. Checking, $45 \times 11 = 495$ and $82 \times 11 = 902$. Summing, we can say that if the outer two digits can be summed to give the middle digit, or if the outer two digits minus the middle digit equals 11, then the number is divisible by 11. Check with 891 and 979.

Eleven is a very interesting number. Square 11 and one gets 121. But this can be arrived at in another way. Imagine an 0 on either side of the 11 so it becomes 0110. If we add each digit to the one next to it we get 0 + 1, 1 + 1, 1 + 0, or 121. Look at 01210. Repeat the process and we get 1331, which is 11^3. Do it again. We get 14641. Here it gets a bit messy, but we can go on. Look at 014641. We get 1, 5, 10, 10, 5, 1. Our last three digits are 051. We can carry our one to the next and we get 1051 with a carry of 1. Therefore we wind up with 161051. If we add the digits, for any power of 11, we get the corresponding power of 2. Like 1 + 5 + 10 + 10 + 5 + 1 = 32, or 2^5. Check a few others for yourself.

Now let's look at two ways of addition of pairs of numbers. We have (a,b) and c,d). Define (a,b)#(c,d) = (ad + bc,bd). For example, (2,3)#(4,7) = (14 + 12,21) or (26,21).(1,2)#(1,2) = (4,4). We can have a rule that we can divide through by a common divisor. Therefore (1,2)#(1,2) = (1,1). Now if we add $\frac{1}{2}$ and $\frac{1}{2}$ we get 1/1 or 1. Similarly if we add 2/3 and 4/7 we get 26/21 se we can see that our operation is the same thing as adding fractions. Hence, fractions can be regarded as pairs of whole numbers and can be operated with as such. (a,b)times (c,d) would be defined as equaling (ac,bd).

We'll define the following kind of addition as follows: (a,b)*(c,d) = (a+c,b+d). (2,3)*(4,7) = (6,10). (2,-3)*(4,7) = (6,4).

A problem can be derived. Let the first term represent going north versus going south. A positive value is north and a negative value is south. Let the second term represent going east versus going west. A positive number is east and a negative number is west. The problem is as follows. A man walks 15 miles north and 6 miles east. Then he walks 5 miles south and 18 miles west. By our notation we can represent the first walk as (15,6) and the second as (-5,-18). Applying our method of addition we get (15,6)*(-5,-18) = (10,-12). This tells us that the man wound up 10 miles north and 12 miles west. This checks because he went 15 miles north and 5 miles south. Then he went also 6 miles east and 18 miles west. The result is 10 miles north and 12 miles west. This is called <u>vector</u> addition.

First goes to A.
Winds up at B.